MATH 147 QUIZ 9 SOLUTIONS

1. Calculate $\iiint_B 3x + y + z^2 \; dV$ where B is the solid parallelopiped spanned by the vectors $v_1 = (1,1,1), v_2 = (1,2,1), v_3 = (2,2,1)$. (5 Points)

   Note that the corresponding transformation from the unit cube to the given parallelopiped is $F(u,v,w) = (u+v+2w, u+2v+2w, u+v+w)$. The Jacobian of this transformation is given by

$$J_F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix} = |-1| = 1.$$

   We perform the change of variables to get

$$\iiint_B 3x + y + z^2 \; dV = \int_0^1 \int_0^1 \int_0^1 3(u+v+2w) + (u+2v+2w) + (u+v+w)^2 \cdot \mathbf{J}_F \cdot dudvdw$$
$$= 1 \cdot \int_0^1 \int_0^1 \int_0^1 3u + 3v + 6w + u + 2v + 2w + u^2 + v^2 + w^2 + 2uv + 2vw + 2uw \; dudvdw$$
$$= \int_0^1 \int_0^1 \int_0^1 4u + u^2 + 2uv + 2uw + 5v + v^2 + 8w + w^2 + 2vw \; dudvdw$$
$$= \int_0^1 \int_0^1 7/3 + 6v + v^2 + 2vw + 9w + w^2 \; dvdw$$
$$= \int_0^1 7/3 + 3 + 1/3 + 10w + w^2 \; dw = 8/3 + 3 + 5 + 1/3 = 11.$$

2. Find the unit tangent vector for $r(t) = (6, \cos(3t), 3\sin(4t))$ for $0 \leq t \leq 2\pi$. (5 points)

   A tangent vector $r'(t)$ is given by $r'(t) = (0, -3\sin(3t), 12\cos(4t))$. We find the magnitude $\|r'(t)\| = \sqrt{0^2 + (-3\sin(3t))^2 + (12\cos(4t))^2} = \sqrt{9\sin^2(3t) + 144\cos^2(4t)}$. Combining we get the unit tangent vector

$$\frac{r'(t)}{\|r'(t)\|} = \left(0, \frac{-3\sin(3t)}{\sqrt{9\sin^2(3t) + 144\cos^2(4t)}}, \frac{12\cos(4t)}{\sqrt{9\sin^2(3t) + 144\cos^2(4t)}}\right).$$